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Indirect Robust Adaptive Fault-Tolerant Control for Attitude Tracking of Spacecraft

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Reliable and cost-effective control of spacecraft should account for modeling uncertainties, unexpected disturbances, subsystem failures, and limited resources simultaneously. This paper presents an indirect (nonregressor-based) approach to attitude tracking control of spacecraft. It is shown that the control algorithms developed are not only robust against external disturbances and adaptive to unknown and time-varying mass/inertia properties, but also able to accommodate actuator failures under limited thrusts. All are achieved with inexpensive online computations (a feature of practical importance in reducing the usage of onboard resources in terms of computing power and memory size). Furthermore, this method is user/designer friendly in that it does not involve a time-consuming design procedure and demands little redesigning or reprogramming during vehicle operation. The benefits of the proposed control method are analytically authenticated and also validated via simulation study.

I. Introduction

ADVANCED space missions demand the development of effective spacecraft control systems to ensure rapid, accurate, and global response to various attitude maneuvering commands. Such a response should be achieved globally in the presence of modeling uncertainties, external disturbances, subsystem failures, and/or limited resources (energy, memory space, computing power, etc.). A significant challenge arises when those issues are treated simultaneously. Although there are rich results on attitude control of spacecraft in the literature, such as nonlinear feedback control [1,2], variable structure control [3], optimal control [4,5], adaptive control, and robust control [6–11], very few have explicitly dealt with these issues concurrently.

In this paper, we present a control scheme to achieve high precision attitude tracking of spacecraft with explicit consideration of uncertainties, disturbances, actuator failures, and limited resources (with particular attention to limited thrusts and cost-effective online computation). The main contributions of this work relative to others are as follows. Indirect (nonregressor-based) robust adaptive control algorithms are developed to achieve robustness against modeling uncertainties and unexpected disturbances. The proposed control scheme ensures stable attitude tracking with guaranteed performance (not just stabilization or rest-to-rest regulation). The method is also adaptive to unknown and time-varying mass/inertia property arising from varying mass distribution due to fuel usage and articulation. Unlike most other four-parameter quaternion-based methods, which demand somewhat heavy online computations [2,4,5,10–12], achieve angular velocity tracking only [13,14], or involve restrictive control parameters [8,15], the proposed method ensures both velocity and attitude tracking with simple design procedures and inexpensive online computations, a feature of practical importance for real-time implementation especially when onboard memory space and computing power are limited. Furthermore, the proposed control scheme is fault tolerant to thruster failures under limited thrusts.

The rest of the paper is organized as follows. In Sec. II, the idea of using indirect core information of the system uncertainties is described and the attitude tracking control problem of spacecraft is formulated using the unit quaternion to represent the attitude orientation. In Sec. III, we present the attitude tracking control algorithms and show their effectiveness in dealing with modeling uncertainties, external disturbances, actuator failures, and thrust saturation. Numerical simulations on various thruster failure situations are presented in Sec. IV to demonstrate the performance of the proposed control method. Finally, we conclude the paper in Sec. V.

II. Attitude Dynamics and Problem Formulation

The spacecraft is modeled as a rigid body with actuators to provide torques along three mutually perpendicular axes that define a body-fixed frame $\mathcal{B}$. The equations of motion in terms of kinematics and dynamics are given by (see [16], Chapter 4):

$$J(\cdot)\dot{\omega} = -\omega^{\times}J(\cdot)\omega + \tau + T_d(\cdot) \tag{1}$$

$$\dot{q} = \tfrac{1}{2}(q^{\times} + q_0 I)\omega \tag{2}$$

$$\dot{q}_0 = -\tfrac{1}{2}q^T\omega \tag{3}$$

where $\omega \in \mathbb{R}^3$ is the angular velocity of the spacecraft with respect to an inertial frame $\mathcal{I}$ and expressed in body frame $\mathcal{B}$; $I \in \mathbb{R}^{3\times3}$ is the identity matrix; $(q, q_0) \in \mathbb{R}^3 \times \mathbb{R}$ denotes the unit quaternion representing the attitude orientation of the spacecraft in the body frame $\mathcal{B}$ with respect to the inertial frame $\mathcal{I}$, satisfying $q^T q + q_0^2 = 1$; $J(\cdot) \in \mathbb{R}^{3\times3}$ is the inertia matrix of the spacecraft expressed in $\mathcal{B}$; $\tau \in \mathbb{R}^3$ and $T_d(\cdot) \in \mathbb{R}^3$ denote the control torques and the external disturbances respectively; and the operator $\omega^{\times}$ denotes a skew-symmetric matrix acting on the vector $\omega = [\omega_1 \quad \omega_2 \quad \omega_3]^T$ and has the form

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$$\omega^\times = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{4}$$

Remark 1: Note that during operation the mass properties of the spacecraft may be uncertain or may change due to onboard payload motion, rotation of solar arrays, or fuel consumptions, making $J(\cdot)$ time varying and even uncertain; thus, control design should not use such $J(\cdot)$ directly. On the other hand, although unknown and time varying, $J(\cdot)$ remains positive definite and bounded during the entire operation of the vehicle. It is therefore reasonable to assume that there exists a constant (unknown) $c_J \geq 0$ such that $\|J(\cdot)\| \leq c_J < \infty$. Also, it is practical to assume that $\|\frac{dJ(\cdot)}{dt}\| \leq c_f < \infty$ for some unknown constant $c_f \geq 0$.

To address the attitude tracking issue, we define the attitude tracking error $(q_e, q_{e0}) \in \mathbb{R}^3 \times \mathbb{R}$ as the relative orientation between the body frame $\mathcal{B}$ and the desired frame $\mathcal{D}$ with orientation $(q_d, q_{d0}) \in \mathbb{R}^3 \times \mathbb{R}$, which is computed as

$$q_e = q_{d0} q - q_0 q_d + q^\times q_d \tag{5}$$

$$q_{e0} = q_{d0} q_0 + q_d^T q \tag{6}$$

where $q_d^T q_d + q_{d0}^2 = 1$ and $q_e^T q_e + q_{e0}^2 = 1$. Assume that $(\omega_d, q_d) \in \mathbb{R}^3 \times \mathbb{R}^3$ (desired angular velocity and attitude) are bounded such that $\|\omega_d\| \leq c_1 < \infty$. $\|\dot{\omega}_d\| \leq c_2 < \infty$ for some unknown constants $c_1 \geq 0$ and $c_2 \geq 0$. The rotation matrix is given by $C = (q_{e0}^2 - q_e^T q_e) I + 2 q_e q_e^T - 2 q_{e0} q_e^\times$ (note that $\|C\| = 1$ and $\dot{C} = -\omega_e^\times C$). Define the relative angular velocity $\omega_e \in \mathbb{R}^3$ of $\mathcal{B}$ with respect to $\mathcal{D}$ as

$$\omega_e = \omega - C\omega_d \tag{7}$$

From Eqs. (1–7), it can be derived that [8,16]

$$J(\cdot)\dot{\omega}_e = -\omega^\times J(\cdot)\omega + \tau + T_d(\cdot) + J(\cdot)(\omega_e^\times C\omega_d - C\dot{\omega}_d) \tag{8}$$

$$\dot{q}_e = \tfrac{1}{2}(q_e^\times + q_{e0} I)\omega_e \tag{9}$$

$$\dot{q}_{e0} = -\tfrac{1}{2} q_e^T \omega_e \tag{10}$$

To develop the control scheme, we define the filtered error variable

$$s = \omega_e + \beta q_e (\beta > 0) \tag{11}$$

where β is a free design parameter chosen by the designer/user. Then, considering Eqs. (7) and (8), we have

$$J(\cdot)\dot{s} = \tau + L(\cdot) - \beta q_e - 0.5 \frac{dJ(\cdot)}{dt} s \tag{12}$$

with

$$L(\cdot) = -\omega^\times J(\cdot)\omega + J(\cdot)[(\omega - C\omega_d)^\times C\omega_d - C\dot{\omega}_d] + T_d(\cdot) + \frac{\beta}{2} J(\cdot)(q_e^\times + q_{e0} I)\omega_e + \beta q_e + 0.5 \frac{dJ(\cdot)}{dt} s \tag{13}$$

The reason for subtracting $\beta q_e + 0.5 \frac{dJ(\cdot)}{dt}$ from Eq. (12) and adding it to Eq. (13) is to facilitate the stability analysis, as will become clear later. Note that $L(\cdot)$ is the lumped term containing two parts: 1) system nonlinearities (depending on desired attitude trajectory and physical parameters, the moment inertia in particular), and 2) external disturbances (changing with operating conditions). One of the major challenges for attitude tracking control design stems from such an uncertain term in the dynamic equation. Different means of treating $L(\cdot)$ lead to different control schemes. Model-based nonlinear control assumes that precise $L(\cdot)$ is available for direct cancellation. Regressor-based adaptive and robust control is based on precise analytic structure of $L(\cdot)$ to derive online updating algorithms for unknown but constant parameters in the system. In general, this method demands heavy computations and time-consuming design procedures, yet cannot deal with time-varying parameters [5–8]. We explore an indirect (nonregressor-based) approach to dealing with the effect of such $L(\cdot)$. This is done by not focusing on $L(\cdot)$ itself, but on its bound, which allows the core information on $L(\cdot)$ to be extracted and used for control design. A similar idea was used in Singh [17] for attitude tracking control without considering singularities, thrusters failures, or thrust limits. In our design, we use this nonregressor-based method to develop robust adaptive control algorithms to cope with the effect of uncertainties and disturbances and to accommodate actuation failures and thrust limits. To this end, we first examine the environmental disturbances due to gravitation, solar radiation, magnetic forces (all could be assumed bounded), and aerodynamic drags (proportional to the square of angular velocity) [16,18]. With all those disturbances considered, it is reasonable to assume that the external disturbances $T_d(\cdot)$ satisfy $\|T_d(\cdot)\| \leq c_g + c_d\|\omega\|^2$ with $c_d \geq 0$ and $c_g \geq 0$ unknown but constant. Also note that $\|q_e^\times + q_{e0} I\| = 1$ and $\|C\| = 1$. Then under the assumption that ω_d, $\dot{\omega}_d$, q_d, and $\dot{q}_d$ are bounded for all $t \geq 0$, it is straightforward to show that $\|(q_e^\times + q_{e0} I)\omega_e\| = \|(q_e^\times + q_{e0} I)(\omega - C\omega_d)\| \leq \|\omega\| + c_w$ for an unknown constant $c_w \geq 0$. Finally, note that $\|\frac{dJ(\cdot)}{dt} s\| \leq c_f \|(\omega - C\omega_d) + \beta q_e\| \leq c_f\|\omega\| + c_0$ for some constant $c_0 \geq 0$. Consequently, it can concluded that, although $L(\cdot)$ contains nonlinear, uncertain, and time-varying terms, there always exist some unknown constants $b_0 \geq 0$, $b_1 \geq 0$, $b_2 \geq 0$, and $b \geq 0$ such that

$$\|L(\cdot)\| \leq b_0 + b_1\|\omega\| + b_3\|\omega\|^2 \leq b\Phi \tag{14}$$

$$\Phi = 1 + \|\omega\| + \|\omega\|^2 \tag{15}$$

It is important to note that Eq. (14) holds for any type of rigid spacecraft and Eq. (15) is independent of any physical parameters or operating conditions of spacecraft under consideration. In other words, both Eqs. (14) and (15) are true regardless of external disturbances, uncertain and time-varying moment inertia, or type of spacecraft. This fact is used for our control design, which leads to a highly robust, adaptive, and fault-tolerant control scheme that does not need to be redesigned or reprogrammed even if the operating conditions or physical parameters of the spacecraft change. Furthermore, using the indirect nonregressor-based method leads to control algorithms that are much simpler in structure and demand much less online computations. More about this is discussed in the next section. The control objective is to develop a robust, adaptive, and fault-tolerant attitude tracking control scheme such that the following goals are achieved in the presence of external disturbances, moment inertia uncertainties, possible actuation failures, and thrust limits:

R1) The closed-loop system is globally stable in that all the internal signal variables are bounded and continuous.

R2) The attitude orientation and velocity tracking errors converge to a small set containing the origin, that is, $\|q_e(t)\| \leq \varepsilon_1^*$, $\|\omega_e(t)\| \leq \varepsilon_2^*$ $t \geq T$.

R3) The tracking performance index

$$I_q = \lim_{t\to\infty} \frac{1}{t} \int_0^t \|s(\omega_e, q_e)\|^2 \, d\tau$$

is bounded.

III. Fault-Tolerant Robust Adaptive Attitude Tracking Control

We first consider the case in which the vehicle is endowed with only three thrusters and each thruster experiences fading actuation (partial power loss) but is still active. The results are extended to the case in which some of the thrusters have completely failed and there exists actuation limit on each thruster.

A. Fault-Tolerant Attitude Tracking Control with Fading Actuators

When the vehicle has three actuators and each of them partially loses its actuation effectiveness, the attitude dynamics are governed by

$$J(\cdot)\dot{\omega} = -\omega^{\times} J(\cdot)\omega + \Gamma(\cdot)\tau + T_d(\cdot) \tag{16}$$

where $\Gamma(\cdot) \in \mathbb{R}^{3\times 3}$ is the actuation effectiveness matrix of the form

$$\Gamma(\cdot) = \begin{bmatrix} \delta_1(\cdot) & 0 & 0 \\ 0 & \delta_2(\cdot) & 0 \\ 0 & 0 & \delta_3(\cdot) \end{bmatrix} \tag{17}$$

with $0 \le \delta_i(\cdot) \le 1$ being the "actuator health indicator" for the ith actuator. The case in which $\delta_i(\cdot) = 1$ implies that the actuator is healthy; $\delta_i(\cdot) = 0$ is the case in which the ith actuator is turned off or totally failed; and $0 < \delta_i(\cdot) < 1$ corresponds to the case in which the ith actuator partially loses its actuating power (fading actuation). When the three actuators encounter partial power loss (fading actuation), the actuation effectiveness matrix $\Gamma(\cdot)$ becomes uncertain or even time varying but remains positive definite. In such case, we have the following result.

Theorem 1: Consider a spacecraft involving fading actuation in that the attitude dynamics are governed by Eq. (16) with $0 < \delta_i(\cdot) < 1$ $(i = 1, 2, 3)$. If the following control scheme is implemented,

$$\tau = -[k_0 + \kappa(t)]s, \qquad \kappa(t) = \frac{\hat{b}\Phi}{\|s\| + \varepsilon}$$
$$\dot{\hat{b}} = -\sigma_1 \hat{b} + \sigma_2 \frac{\|s\|^2 \Phi}{\|s\| + \varepsilon}, \qquad \varepsilon = \frac{\mu}{1 + \Phi} \tag{18}$$

where $k_0 > 0$, $\mu > 0$, $\sigma_1 > 0$, and $\sigma_2 > 0$ are chosen by the designer. Then the control objectives as stated in R1–R3 are achieved.

Proof: Let $\lambda_{\min} \le \min\{\delta_1, \delta_2, \delta_3\}$, which is positive as long as the thrust power is not completely lost. Consider the Lyapunov function candidate

$$V = \frac{1}{2} s^T J(\cdot) s + \frac{1}{2\sigma_2 \lambda_{\min}} (b - \lambda_{\min}\hat{b})^2 + \beta\Big[q_e^T q_e + (1 - q_{e0})^2\Big] \tag{19}$$

In view of Eq. (12), it follows that

$$\dot{V} = 0.5 s^T \frac{\mathrm{d}J(\cdot)}{\mathrm{d}t} s^T + s^T\left(\tau + L - 0.5 \frac{\mathrm{d}J(\cdot)}{\mathrm{d}t} s - \beta q_e\right) - \frac{1}{\sigma_2}(b - \lambda_{\min}\hat{b})\dot{\hat{b}} + \beta\omega_e^T q_e \tag{20}$$

Note that the last term in Eq. (20) was obtained by using $q_e^T q_e + q_{e0}^2 = 1$, $\dot{q}_{e0} = -\frac{1}{2} q_e^T \omega_e$, and the definition of s as in Eq. (11). Further simplification of Eq. (20) leads to

$$\dot{V} = s^T(\tau + L) - \tfrac{1}{\sigma_2}(b - \lambda_{\min}\hat{b})\dot{\hat{b}} - \beta^2 q_e^T q_e$$

With the control algorithms as given in Eq. (18), it can be shown that

$$\begin{aligned}
\dot{V} &= -s^T\Gamma(k_0 + \kappa)s + s^T L + \frac{1}{\sigma_2}(b - \lambda_{\min}\hat{b})(-\dot{\hat{b}}) - \beta^2 q_e^T q_e \\
&\le -k_0\lambda_{\min} s^T s - \beta^2 q_e^T q_e - \lambda_{\min}\kappa\|s\|^2 + \|s\| b\Phi \\
&\quad + \frac{1}{\sigma_2}(b - \lambda_{\min}\hat{b})(-\dot{\hat{b}}) = -k_0\lambda_{\min} s^T s - \beta^2 q_e^T q_e \\
&\quad - \frac{\lambda_{\min}\hat{b}\Phi}{\|s\| + \varepsilon}\|s\|^2 + \|s\| b\Phi + \frac{1}{\sigma_2}(b - \lambda_{\min}\hat{b})(-\dot{\hat{b}}) \\
&\le -k_0\lambda_{\min} s^T s - \beta^2 q_e^T q_e + (b - \lambda_{\min}\hat{b})\frac{\|s\|^2\Phi}{\|s\| + \varepsilon} \\
&\quad + \varepsilon b\Phi + \frac{1}{\sigma_2}(b - \lambda_{\min}\hat{b})(-\dot{\hat{b}}) \\
&\le -k_0\lambda_{\min} s^T s - \beta^2 q_e^T q_e + \mu b + \frac{\sigma_1}{\sigma_2}(b - \lambda_{\min}\hat{b})\hat{b}
\end{aligned} \tag{21}$$

where $\frac{\|s\|}{\|s\|+\varepsilon} \le 1$ $(\forall\ \varepsilon \ge 0)$, $\frac{\Phi}{1+\Phi} \le 1$, $(\forall\ \Phi \ge 0)$, and the updating scheme for $\hat{b}$ as given in Eq. (18) were employed. Because

$$\frac{\sigma_1}{\sigma_2}(b - \lambda_{\min}\hat{b})\hat{b} = -\frac{\sigma_1}{2\sigma_2\lambda_{\min}}(b - \lambda_{\min}\hat{b})^2 + \frac{\sigma_1}{2\sigma_2\lambda_{\min}}\left(b^2 - \lambda_{\min}^2\hat{b}^2\right) \le -\frac{\sigma_1}{2\sigma_2\lambda_{\min}}(b - \lambda_{\min}\hat{b})^2 + \frac{\sigma_1 b^2}{2\sigma_2\lambda_{\min}}$$

and

$$\frac{\sigma_1}{\sigma_2}(b - \lambda_{\min}\hat{b})\hat{b} = \frac{\sigma_1\lambda_{\min}}{\sigma_2}\left(\frac{b\hat{b}}{\lambda_{\min}} - \hat{b}^2\right) = -\frac{\sigma_1\lambda_{\min}}{\sigma_2}\left(\hat{b} - \frac{b}{2\lambda_{\min}}\right)^2 + \frac{\sigma_1 b^2}{4\sigma_2\lambda_{\min}} \le \frac{\sigma_1 b^2}{4\sigma_2\lambda_{\min}}$$

It is readily obtained from Eq. (19) that

$$\dot{V} \le -k_0\lambda_{\min} s^T s - \beta^2 q_e^T q_e - \frac{\sigma_1}{2\sigma_2\lambda_{\min}}(b - \lambda_{\min}\hat{b})^2 + \frac{\sigma_1 b^2}{2\sigma_2\lambda_{\min}} + b\mu \le -\lambda_0 V + \varepsilon_0 \tag{22a}$$

$$\dot{V} \le -k_0\lambda_{\min} s^T s - \beta^2 q_e^T q_e + \varepsilon_1 \tag{22b}$$

$$\dot{V} \le -k_0\lambda_{\min} s^T s - \beta^2 q_e^T q_e + \varepsilon_2 \tag{22c}$$

with

$$\lambda_0 = \min\{k_0\lambda_{\min}/\lambda_{\max}(J), \sigma_1, \beta\} > 0$$
$$\varepsilon_0 = \beta(1 - q_0)^2 + \frac{\sigma_1 b^2}{2\sigma_2\lambda_{\min}} + b\mu < \infty$$
$$\varepsilon_1 = \frac{\sigma_1 b^2}{2\sigma_2\lambda_{\min}} + b\mu < \infty, \qquad \varepsilon_2 = \frac{\sigma_1 b^2}{4\sigma_2\lambda_{\min}} + b\mu < \varepsilon_1 < \infty$$

Note that Eq. (22a) implies that $V \in \mathcal{L}_\infty$. Consequently, we have $s \in \mathcal{L}_\infty$, $q_e \in \mathcal{L}_\infty$, and $\hat{b} \in \mathcal{L}_\infty$, which ensure that $\Phi \in \mathcal{L}_\infty$, $\dot{\hat{b}} \in \mathcal{L}_\infty$, $\kappa \in \mathcal{L}_\infty$, and $F_c \in \mathcal{L}_\infty$ on the time interval $[0, T]$ and the solution can be extended past $t = T$. The same arguments hold for any finite T and the solution exists for all $t \in [0, \infty)$ [15,19]. Moreover, it is seen from Eqs. (22b) and (22c) that $\dot{V} < 0$ when (s, q_e) are outside of the set

$$B_1 \triangleq \left\{(s, q_e)\colon \|s\| \le \sqrt{\frac{\varepsilon_1}{k_0\lambda_{\min}}}, \|q_e\| \le \frac{\sqrt{\varepsilon_1}}{\beta}\right\}$$

or the set

$$B_2 \triangleq \left\{(s, q_e)\colon \|s\| \le \sqrt{\frac{\varepsilon_2}{k_0\lambda_{\min}}}, \|q_e\| \le \frac{\sqrt{\varepsilon_2}}{\beta}\right\}$$

Because $\varepsilon_1 > \varepsilon_2$, set B_1 encloses set B_2 completely; thus, the system states (s, q_e) may move in or out of B_2 but, once inside the set B_1, (s, q_e) cannot go out of it because they will be attracted back to B_2. That is, the states are confined in the set B_1. Furthermore, because of Eq. (11), ω_e is bounded as well. Its bound can be determined as

$$B_3 \triangleq \{\omega_e\colon \|\omega_e\| \le \sqrt{\varepsilon_2}\}$$

This can be shown by using $\omega_e + 0.5\frac{\mathrm{d}J(\cdot)}{\mathrm{d}t}$, instead of $\beta q_e + 0.5\frac{\mathrm{d}J(\cdot)}{\mathrm{d}t}$, in Eqs. (12) and (13). With the same Lyapunov function candidate as given in Eq. (19), it follows that

$$\dot V = 0.5 s^T \frac{\mathrm{d}J(\cdot)}{\mathrm{d}t} s^T + s^T\left(\tau + L_\omega - 0.5\frac{\mathrm{d}J(\cdot)}{\mathrm{d}t}s - \omega_e\right) - \frac{1}{\sigma_2}(b - \lambda_{\min}\hat b)\dot{\hat b} + \beta\omega_e^T q_e = s^T\tau + s^T L_\omega - \frac{1}{\sigma_2}(b - \lambda_{\min}\hat b)\dot{\hat b} - \omega_e^T\omega_e$$

where $L_\omega(\cdot)$ takes the same form as in Eq. (13) with $\beta q_e + 0.5\frac{\mathrm{d}J(\cdot)}{\mathrm{d}t}$ being replaced by $\omega_e + 0.5\frac{\mathrm{d}J(\cdot)}{\mathrm{d}t}$. Note that such $L_\omega(\cdot)$ still obeys the relation (14) and (15). Thus, it is straightforward to show that $\dot V \le -k_0\lambda_{\min}s^T s - \omega_e^T\omega_e + \varepsilon_2$, that is., ω_e is bounded by

$$B_3 \triangleq \{\omega_e\colon \|\omega_e\| \le \sqrt{\varepsilon_2}\}$$

Finally, it can be shown from Eq. (22c) that

$$\lim_{t\to\infty}\frac{1}{t}\int_0^t \|s\|^2\,\mathrm{d}t \le \lim_{t\to\infty}\left(\frac{V(0)}{k_0\lambda_{\min}t} + \frac{\varepsilon_2}{k_0\lambda_{\min}}\right) \le \frac{\sigma_1 b^2}{4\sigma_2\lambda_{\min}^2 k_0} + \frac{b\mu}{k_0\lambda_{\min}} < \infty$$

implying that larger k_0 and σ_2 (faster updating rate) and smaller μ and σ_1 lead to better performance. Thus, the result as stated in Theorem 1 is established.

Remark 2: Note that when all the actuators are healthy, the actuation effectiveness matrix $\Gamma(\cdot) = I$ and the control scheme remains the same as in Eq. (18), implying that the proposed control as given in Eq. (18) is able to achieve attitude tracking regardless of the thruster health condition as long as no thruster is completely failed.

Remark 3: It is worth mentioning that, although the parameter $\lambda_{\min}$ is used in stability analysis, an analytical estimate of this parameter is not needed because the proposed control algorithms do not involve such a parameter. As can be seen from Eq. (18), the control scheme only involves simple functions and variables. All can be computed inexpensively; less computing power is needed compared to most existing methods.

If a vehicle with only three actuators suffers from a failure of one or more thrusters ($\delta_i = 0$), the vehicle becomes underactuated. Under certain conditions, it is still possible to achieve attitude tracking along certain directions [20,21]. It is noted that in this case arbitrary attitude tracking along all three directions in the presence of modeling uncertainties and disturbances as well as thrust faults becomes physically challenging. We address this issue next by considering the case in which the vehicle is equipped with more than three thrusters.

B. Fault-Tolerant Attitude Tracking Control Under Engine Failures

When the vehicle has more than three thrusters, some of which might experience losing power partially or totally, its attitude dynamics are described by

$$J(\cdot)\dot\omega = -\omega^\times J(\cdot)\omega + D\Gamma(\cdot)F + T_d(\cdot) \tag{23}$$

where $F \in \mathbb{R}^n$ ($n > 3$) denotes the propulsion force vector produced by the n thrusters, $D \in \mathbb{R}^{3\times n}$ is the thruster distribution matrix (for a given vehicle, D is available and can be made full-row rank by properly placing the thrusters at certain locations and directions on the vehicle), and the matrix $\Gamma(\cdot) = \mathrm{diag}(\delta_1, \ldots, \delta_n) \in \mathbb{R}^{n\times n}$ is a matrix characterizing the health condition of the engines. $\Gamma(\cdot)$ is an identity matrix *only* if all engines are healthy. Otherwise, $\Gamma(\cdot)$ is a diagonal matrix with uncertain and time-varying diagonal elements. The case of $\delta_i = 0$ indicates that the engine has either totally failed or been shut down purposely and, in either case, we want to keep the vehicle under control and achieve stable and high precise attitude tracking. It is interesting that this can be achieved with a slight modification to the control scheme presented earlier, as follows:

$$F = -[k_0 + \kappa(t)]D^T s, \qquad \kappa(t) = \frac{\hat b\Phi}{\|s\| + \varepsilon}$$
$$\dot{\hat b} = -\sigma_1\hat b + \sigma_2\frac{\|s\|^2\Phi}{\|s\| + \varepsilon}, \qquad \varepsilon = \frac{\mu}{1 + \Phi} \tag{24}$$

where all the control variables and parameters are defined as before. The stability and control performance can be analyzed by using a similar Lyapunov function candidate:

$$V = \frac{1}{2}s^T J(.)s + \frac{1}{2\sigma_2\lambda_{\min}}(b - \lambda_{\min}\hat b)^2 + \beta\Big[q_e^T q_e + (1 - q_{e0})^2\Big] \tag{25}$$

where $\lambda_{\min} > 0$ is some unknown constant less than the minimum eigenvalue of $D\Gamma(\cdot)D^T$. Following the same lines as in the proof of Theorem 1, it can be shown that

$$\dot V = -s^T(k_0 + \kappa)D\Gamma D^T s + s^T L + \frac{1}{\sigma_2}(b - \lambda_{\min}\hat b)(-\dot{\hat b}) - \beta^2 q_e^T q_e \le -k_0\lambda_{\min}s^T s - \beta^2 q_e^T q_e - \frac{\sigma_1}{2\lambda_{\min}\sigma_2}(b - \lambda_{\min}\hat b)^2 + \frac{\sigma_1 b^2}{2\lambda_{\min}\sigma_2} + b\mu$$

and the following result can be established.

Theorem 2: Consider the spacecraft with the attitude dynamics as in Eq. (23). Assume that there are n thrusters ($n > 3$) properly mounted along the vehicle and the remaining active thrusters (including the fading ones) are able to produce a combined force sufficient enough to allow the vehicle to perform given maneuvers. If the control scheme (24) is applied, stable attitude tracking is ensured when all thrusters of the vehicle suffer from fading actuation ($0 < \delta_i < 1$). Furthermore, the control scheme is still able to achieve stable attitude tracking even if some of the thrusters completely fail to work as long as the number of failed thrusters is no more than $n - 3$ such that $D\Gamma(\cdot)D^T$ remains positive definite.

Remark 4: Because $\Gamma(\cdot)$ is not used in the control scheme, there is no need to include a health monitoring unit to identify or estimate which thruster is unhealthy. Knowledge of the degree of failure for each thruster is not even needed. The thruster fault accommodation/compensation is done automatically and adaptively by the proposed control algorithms. This feature is necessary to build affordable and effective fault-tolerant flight control schemes.

Remark 5: Because the thruster distribution matrix D is made full-row rank (by proper placement of the thrusters on the vehicle), the stability is ensured as long as $D\Gamma(\cdot)D^T$ is positive definite. For instance, consider a vehicle with six thrusters that are distributed in such a way that

$$D = \begin{bmatrix} 1 & 0 & 0 & 0.2 & 0.1 & 0.3 \\ 0 & 1 & 0 & 0.6 & 0.5 & 0.3 \\ 0 & 0 & 1 & 0 & 0.2 & 0.4 \end{bmatrix}$$

and the actuation effectiveness matrix $\Gamma(\cdot) = \mathrm{diag}[0.5, 0.7, 0.7, 0, 0, 0]$, implying that three of the thrusters suffer from fading actuation and the other three have totally failed. In such a case, $\lambda_{\min}(D\Gamma(\cdot)D^T) = 0.5$ and the control scheme is still able to ensure stable attitude tracking. Of course, the implicit assumption is that the remaining active thrusters are able to produce a sufficient actuating torque vector for the vehicle to perform the given maneuvers.

C. Fault-Tolerant Attitude Tracking Control Under Thrust Failures and Thrust Limits

It is of theoretical and practical importance to consider the factors of uncertainties, disturbances, actuation failures, and thrust limits simultaneously. Let $|F_i| \le F_{\max}^i$ ($i = 1, 2, \ldots, n$) and $F_{\max}^i > 0$ denote the maximum allowable thrust force of each thruster. For the system to admit a feasible attitude tracking control solution under such a severe situation, it is necessary to assume that the functional (not necessarily healthy) thrusters are able to produce a combined force sufficient enough to allow the vehicle to perform a given maneuver in the sense that there exists a constant $f_0 > 0$ so that

$$\frac{\lambda_{\min} F_{\max}}{\|D\|} \geq b\Phi + f_0 \tag{26}$$

where $\lambda_{\min} > 0$ is defined as before and $F_{\max} = \min\{F^1_{\max}, F^2_{\max}, \ldots, F^n_{\max}\}$. A similar assumption was proposed in [13,14,18,22] without considering thrust faults. The control scheme ensuring attitude tracking of spacecraft under the conditions as mentioned is given by

$$F = -F_{\max} \frac{D^T}{\|D\|} \mathrm{sat}([k_0 + \kappa(t)]s) \tag{27}$$

with

$$\mathrm{sat}\,([k_0 + \kappa(t)]s) = \begin{cases} \frac{s}{\|s\|} & \text{if } \|s\| \geq F_{\max}/(k_0 + \kappa) \\ \frac{[k_0+\kappa(t)]s}{F_{\max}} & \text{if } \|s\| \leq F_{\max}/(k_0 + \kappa) \end{cases}$$

$$\kappa(t) = \frac{\hat{b}\Phi}{\|s\| + \varepsilon}, \qquad \dot{\hat{b}} = -\sigma_1 \hat{b} + \sigma_2 \frac{\|s\|^2 \Phi}{\|s\| + \varepsilon}, \qquad \varepsilon = \frac{\mu}{1 + \Phi}$$

where $\Phi = 1 + \|\omega\| + \|\omega\|^2$ and $k_0 > 0$, $\mu > 0$, $\sigma_1 > 0$, and $\sigma_2 > 0$ are chosen by the designer.

Theorem 3: Consider the spacecraft equipped with n thrusters ($n > 3$) under thrust limits $|F_i| \leq F^i_{\max}$ ($i = 1, 2, \ldots, n$). Assume the number of the failed thrusters is no more than $n - 3$ such that $D\Gamma(\cdot)D^T$ is positive definite and that the remaining active thrusters (including the fading ones) are able to produce a combined force sufficient enough to allow the vehicle to perform given maneuvers in that Eq. (26) holds. If the control scheme (27) is applied, then the control objectives as stated in R1–R3 are achieved.

To show the stability, two cases need to be addressed.

Case 1: $\|s\| \geq F_{\max}/(k_0 + \kappa)$, $F = -F_{\max} \frac{D^T s}{\|D\|\|s\|}$, which satisfies the force constraint of $|F_i| \leq F_{\max} \leq F^i_{\max}$, considering the Lyapunov function candidate as

$$V = \tfrac{1}{2} s^T J(\cdot) s + \beta\Big[q_e^T q_e + (1 - q_{e0})^2\Big]$$

it follows that

$$\begin{aligned} \dot{V} &= s^T(D\Gamma F + L) - \beta^2 q_e^T q_e \\ &= -s^T \left\{ \frac{D\Gamma D^T F_{\max} s}{\|D\|\|s\|} + L \right\} - \beta^2 q_e^T q_e \\ &\leq -\|s\| \left\{ \frac{\lambda_{\min} F_{\max}}{\|D\|} - b\Phi \right\} - \beta q_e^T q_e \leq -\|s\| f_0 - \beta^2 \|q_e\|^2 < 0 \end{aligned}$$

Case 2: $\|s\| \leq F_{\max}/(k_0 + \kappa)$, $F = -\frac{D^T}{\|D\|}(k_0 + \kappa)s$, considering the Lyapunov function candidate as in Eq. (25), it follows that

$$\begin{aligned} \dot{V} &= -s^T (k_0 + \kappa) \frac{D\Gamma D^T}{\|D\|} s + s^T L \\ &+ \frac{1}{\sigma_2}(b - \lambda_{\min}\hat{b})(-\dot{\hat{b}}) - \beta^2 q_e^T q_e \leq -\frac{k_0 \lambda_{\min}}{\|D\|} s^T s - \beta^2 q_e^T q_e \\ &- \frac{\sigma_1}{2\lambda_{\min}\sigma_2}(b - \lambda_{\min}\hat{b})^2 + \frac{\sigma_1 b^2}{2\lambda_{\min}\sigma_2} + b\mu \end{aligned}$$

The result thus is established using the same argument as in the Proof of Theorem 2. Note that $\mathrm{sat}(\cdot)$ is continuous everywhere including at the point $\|s\| = F_{\max}/(k_0 + \kappa)$, the control is bounded and smooth everywhere. Also, it can be verified that $|F_i| \leq F_{\max} \leq F^i_{\max}$ $\forall\, s$.

Remark 6: It is seen that control scheme (27) not only meets the thrust limit, but also effectively accommodates modeling uncertainties, external disturbances, and thrust faults. It has a simple structure in which the inertia matrix (practically time varying and uncertain) or its estimation is not directly involved, thus simplifying the design process and online computation significantly. The control scheme can be easily set up and implemented with much less demand on onboard computing power and memory space compared to most existing methods.

IV. Simulation Study

To verify the effectiveness of the proposed control scheme, simulations on a vehicle with six thrusters under various conditions are conducted. The six thrusters are assumed to be distributed symmetrically on three axes of the body frame $\mathcal{B}$ of the spacecraft, and the propulsion force is perpendicular to the corresponding axis such that the distribution matrix can be simply determined by the distance l_i ($i = 1, \ldots, 6$) between the center of mass of the vehicle and the position of the thruster, as illustrated in Fig. 1. In this simulation, $l_1 = l_2 = 0.8$ (m), $l_3 = l_4 = l_5 = l_6 = 0.7$ (m), so that the distribution matrix is

$$D = \begin{bmatrix} 0.8 & -0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & -0.7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.7 & -0.7 \end{bmatrix}$$

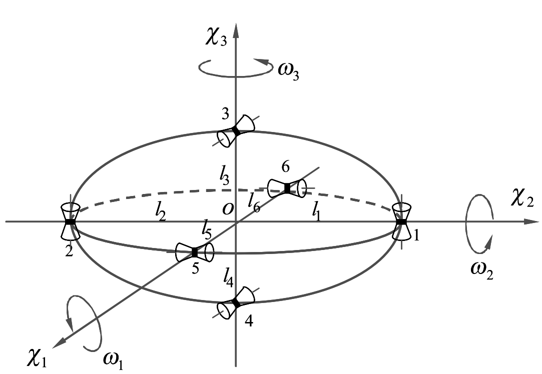
This simulation is carried out under the condition that the moment inertia matrix $J(\cdot) = J_0 + J_u(t)$ is unknown and time varying (due to, for instance, both fuel burning and payload release) as reflected in $J_u(t)$ and depicted in Fig. 2 (noting the "jump" variation of the elements) and the normal part J_0 is

$$J_0 = \begin{bmatrix} 20 & 0 & 0.9 \\ 0 & 17 & 0 \\ 0.9 & 0 & 15 \end{bmatrix}$$

taken from [13,14,22] (J_0 is usually nondiagonal unless the axis of the body frame and the principal axis coincide).

The vehicle is to perform the maneuver that changes its attitude from the initial attitude

$$q(0) = [-0.1, \quad 0.15, \quad -0.2]^T \quad \text{and} \quad q_0(0) = \sqrt{1 - q^T q}$$



Distribution Matrix:

$$D = \begin{bmatrix} l_1 & -l_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & l_3 & -l_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & l_5 & -l_6 \end{bmatrix}$$

Fig. 1 Distribution schematics of the six thrusters and the distribution matrix.

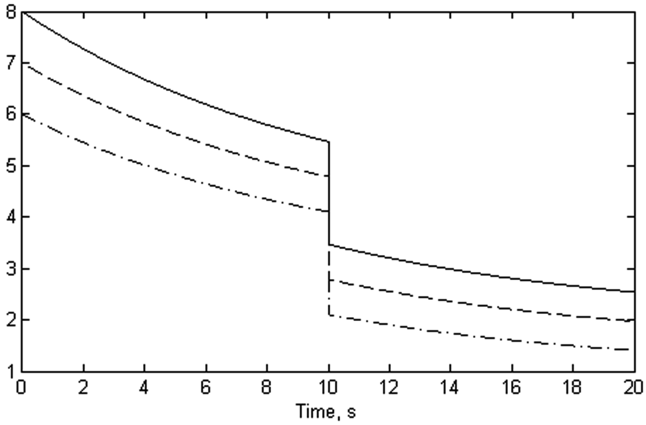


Fig. 2 Uncertain moment of inertia.

to a new attitude along the desired trajectory

$$q_d(t) = \left[\frac{\sqrt{3}}{3}\sin(-0.1t), \frac{\sqrt{6}}{6}\sin(-0.1t)\frac{1}{2}\sin(-0.1t)\right]^T \quad \text{and}$$

$$q_{d0}(t) = \sqrt{1 - q_d^T q_d}$$

The disturbance torque simulated is of the form $T_d = (\|\omega\|^2 + 0.5)[\sin 0.8t \quad \cos 0.5t \quad \cos 0.3t]^T$. We simulate three different cases: 1) healthy thrusters, 2) thrusters with fading and failed actuation, and 3) thrusters with limited thrusts and failed actuation. Our earlier theoretical analysis declares that controller (24) can deal with cases 1 and 2 and controller (27) can handle case 3 effectively; we now verify that claim. Note that, in both controllers, one only needs to simply select the control parameters $k_0 > 0$, $\sigma_2 > 0$, $\beta > 0$, $\sigma_1 > 0$, and $\mu > 0$. In the simulation, they are chosen quite arbitrarily as $k_0 = 20$, $\sigma_2 = 100$, $\beta = 2$, $\sigma_1 = 0.01$, and $\mu = 0.1$ and remain unchanged for all the simulation cases. Also note that there is no need to carry out the time-consuming and sometimes painful task of analytical derivation (in contrast with most other control methods) to set up the controller.

A. Healthy Actuators

With all thrusters functioning healthily, control scheme (24) is applied. The angular velocity and attitude tracking errors are shown in Fig. 3. The three control torques produced by the six thrusters are presented in Fig. 4. The control performance index and the adaptive parameter $\hat{b}(t)$ are depicted in Figs. 5 and 6, respectively. One can observe high control precision and good tracking process. The control action is bounded and smooth.

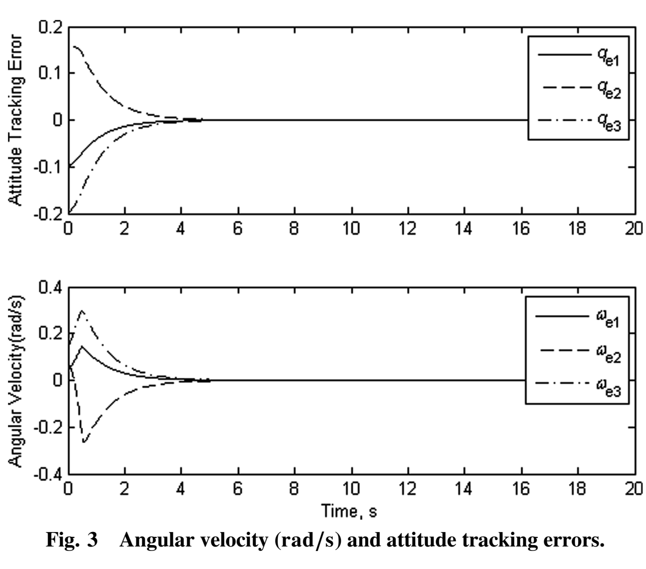


Fig. 3 Angular velocity (rad/s) and attitude tracking errors.

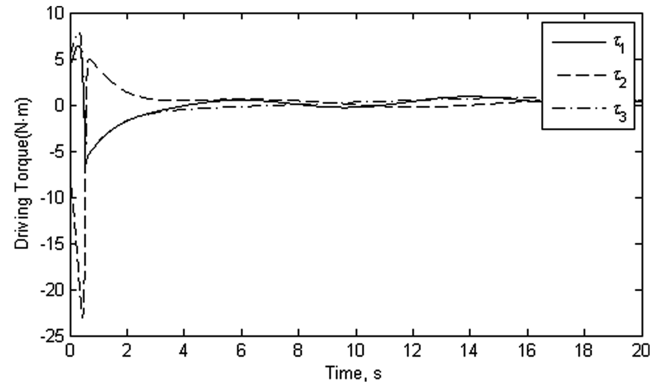


Fig. 4 Three actuation torques (N · m).

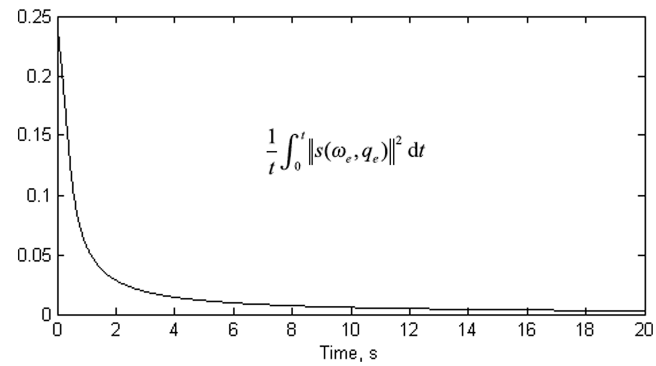


Fig. 5 Bounded control performance index.

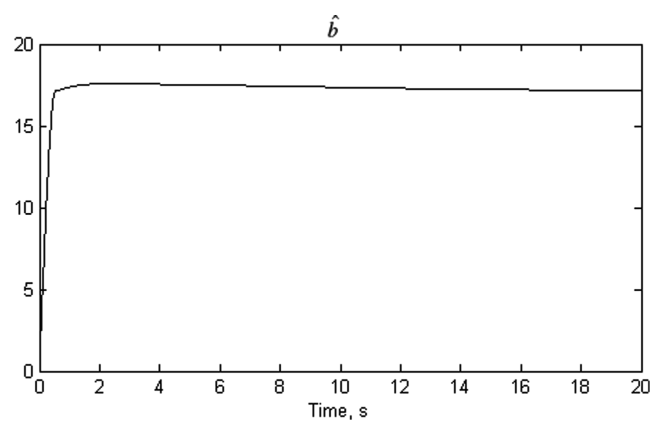


Fig. 6 Adaptive parameter $\hat{b}(t)$.

B. Fading Actuation and Failed Actuation

This represents a severe case in which not only do some thrusters lose partial power with randomly varying heath levels, but some thrusters have also totally failed or are shut down purposely, as illustrated in Fig. 7 in which the health level of each thruster is generated by the following function

$$\delta_i = 0.7 + 0.15\text{rand}(t_i) + 0.1\sin(0.5t + i\pi/3), \qquad (i = 1, \ldots, 6) \tag{28}$$

which swings between 0.95 and 0.45, where $t_i = \text{mod}(t + \Delta t_i, \Delta T)$, with $\Delta t_i = 0.4(i-1)$ s and $\Delta T = 2.4$ s denoting the time delay and generation interval, respectively. rand$(\cdot) \in [-1, 1]$ is a random number generator, which generates a random number if $t_i = 0$ and holds its previous value if $t_i \neq 0$. Among the six thrusters, thruster 3 only supplies 20% of the actuation power at the time instant $t = 8$ s and, after, thrusters 4 and 5 have failed or are shut down after $t = 10$ and 12 s, respectively. The control (24) with the same control

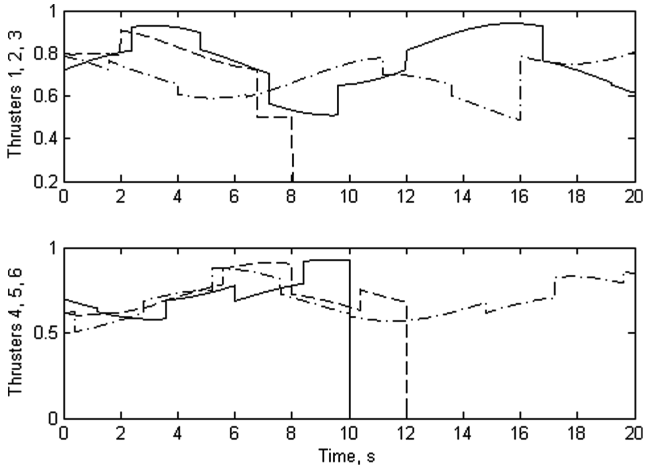


Fig. 7 Health indicator $\Gamma(t)$.

parameters as in the case of healthy thrusters is employed. The angular velocity and attitude tracking errors are presented in Fig. 8. The driving torque is shown in Fig. 9. It is seen that high control precision and good tracking process are still obtained, and the control action is bounded and smooth with no chattering. As long as the thrusting force of each thruster is still within its maximum allowable limit, the control is able to automatically and instantly command/assign a suitable amount of propulsion for each thruster to supply. If the assigned amount reaches beyond the thruster's limit, control (27) has to be used, as shown in Sec. IV.C.

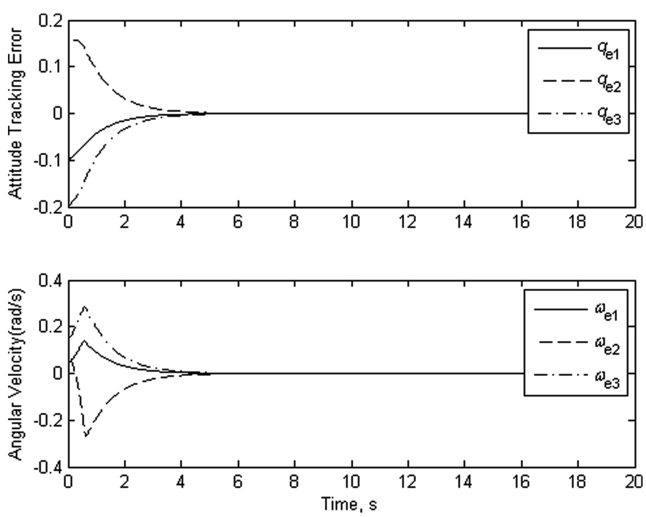


Fig. 8 Angular velocity (rad/s) and attitude tracking errors.

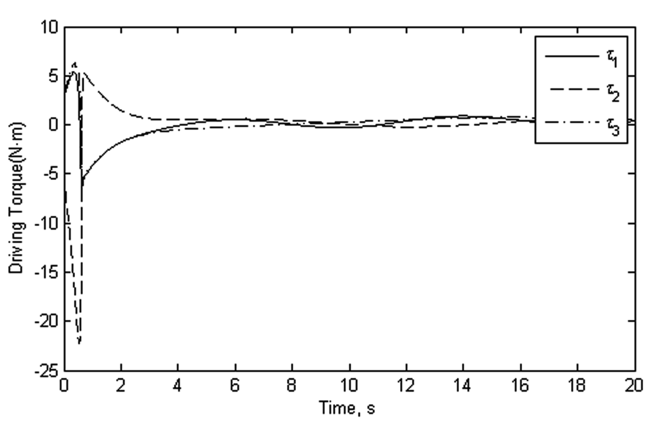


Fig. 9 Three actuation torques (N · m).

C. Actuator Failures and Limited Thrusts

This case not only involves failed thrusters but also limited thrusts, that is, $F_{\max} = 5$ (N) is imposed on all thrusters. The thrusters are assumed to experience the same faults as described in Sec. IV.B. Control scheme (27) is used with the same control parameters as in the case of healthy thrusters. The control performance and the related control signals are presented in Figs. 10–13. It is seen that fairly good control performance is achieved under such severe thruster faults with limited thrust.

The simulations on all those severe cases including the worst one all indicate that the proposed control is indeed robust, adaptive, fault

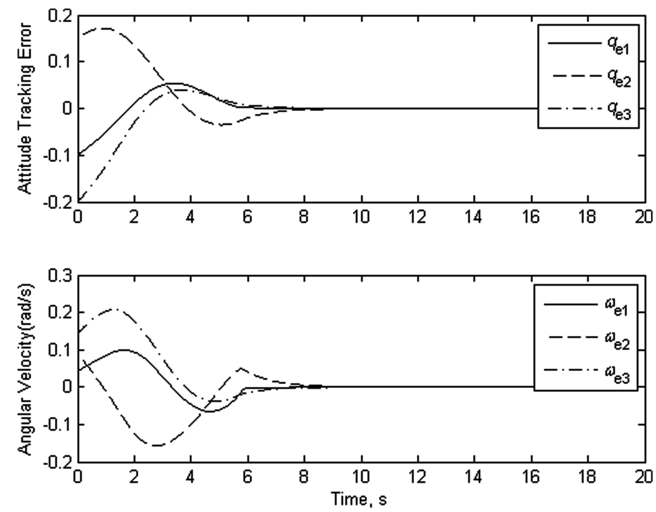


Fig. 10 Angular velocity (rad/s) and attitude tracking errors.

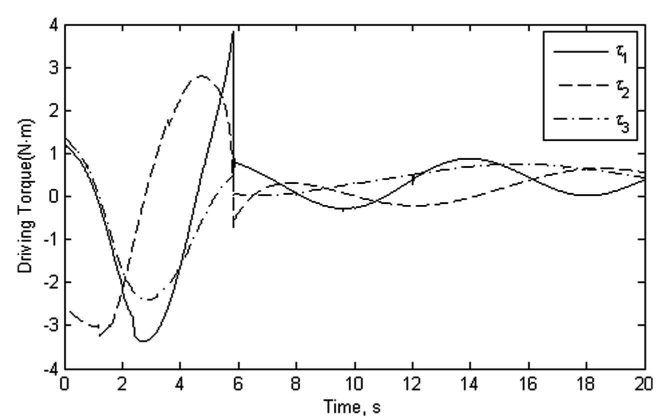


Fig. 11 Three actuation torques (N · m).

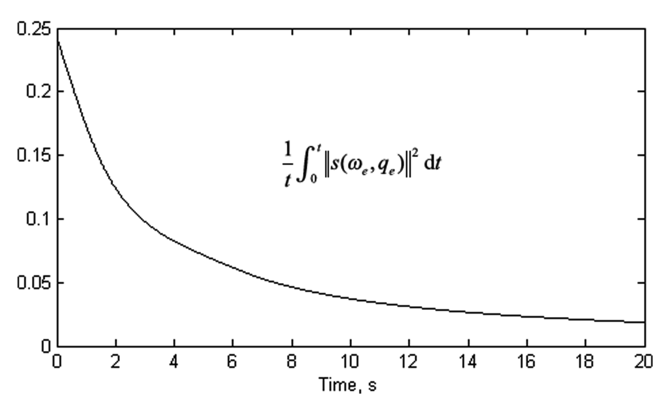


Fig. 12 Bounded control performance index.

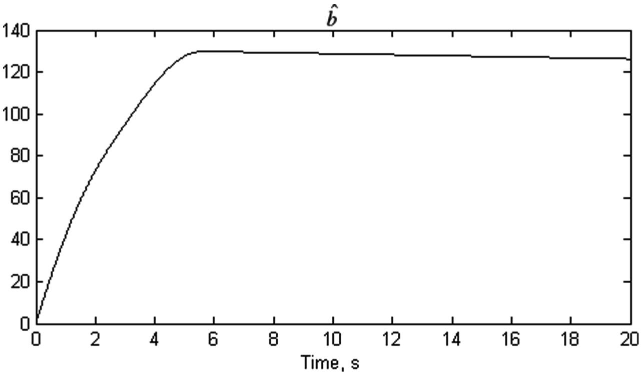


Fig. 13 Adaptive parameter $\hat{b}(t)$.

tolerant, and user/designer friendly and that little reprogramming or redesigning is needed when operating conditions change.

V. Conclusions

Although there is a plethora of attitude tracking control designs published in the literature for rigid spacecraft, very few have addressed robustness, adaptation, fault tolerance, inexpensive computations, and thrust limit (saturation) simultaneously. This work developed a stable attitude tracking control strategy for spacecraft for which external disturbances and modeling uncertainties are effectively attenuated and detail system dynamics are not needed in design. Moreover, the proposed control scheme is able to accommodate actuator failures and does so under thrust limits. It has a simple structure without directly involving the inertia matrix (which is practically time varying and uncertain) or its estimation, thus simplifying the design process and online computation significantly. The implementation of the proposed control scheme demands much less onboard computing power and memory space compared to most existing methods. Numerical simulations on several severe actuation cases including the worst one confirm the claims.

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